

# NEW LOOK AT GEOMETRICAL SCALING

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In this note we discuss emergence of geometrical scaling (firstly proposed for deep inelastic collisions) in pp scattering at the LHC and in heavy ion collisions at RHIC. After discussing general properties of geometrical scaling (GS) we focus on simple signatures of GS, namely on ratios of the  $p_T$  values at which multiplicity spectra for different energies are equal.

## 1 Geometrical Scaling in Deep Inelastic Scattering

One of the outstanding problems of the evolution of parton densities is the rapid increase of the number of gluons at low Bjorken  $x$  [1]. Such growth has to be tamed at some point. The scale at which this happens is called saturation scale  $Q_s$  and it depends on the Bjorken  $x$ . The explicit form of the saturation scale follows from the fact that  $Q_s^2(x)$  is related to the gluon distribution in the proton at low  $x$  [2]:

$$Q_s^2(x) = Q_0^2 (x/x_0)^{-\lambda} \quad (1)$$

where  $Q_0 \sim 1$  GeV and  $x_0 \sim 10^{-3}$  are free parameters whose precise values can be extracted from fits to the HERA data. Power  $\lambda$  is known to be of the order  $\lambda \sim 0.2 \div 0.3$ .

Based on the saturation idea Golec-Biernat and Wüsthoff proposed a model [2] where the reduced cross-section  $\sigma_{\gamma^*p} \sim F_2/Q^2$  is written as:

$$\sigma_{\gamma^*p} = \int dr^2 |\psi(r, Q^2)|^2 \sigma_{dP}(r^2, x) \quad (2)$$

where  $\psi$  is an amplitude for photon dissociation into a  $\bar{q}q$  pair of size  $r$ , which is explicitly known, and  $\sigma_{dP}$  is a model dipole-proton cross-section.

There are two important features of  $\sigma_{dP}$  that are incorporated in the GBW model. Firstly, instead of taking  $\sigma_{dP}$  to depend independently on  $r$  and  $W$  ( $\gamma^*p$  CMS energy) it is assumed that  $\sigma_{dP}$  depends on Bjorken  $x$

$$x = Q^2/(Q^2 + W^2 - M_p^2) \quad (3)$$

where  $M_p$  stands for the proton mass. Furthermore, in the GBW model  $\sigma_{dP}$  depends on  $x$  through the saturation scale  $Q_s(1)$ , namely:

$$\sigma_{dP}(r^2, x) = \sigma_{dP}(r^2 Q_s^2(x)). \quad (4)$$

Given the fact that for massless quarks distribution amplitude  $\psi$  is a function of a product of  $r$  and  $Q$ :

$$|\psi(r, Q^2)|^2 = Q^2 |\tilde{\psi}(rQ)|^2 \quad (5)$$

it follows that

$$\sigma_{\gamma^*p} = \text{function}(Q^2/Q_s^2(x)) \quad (6)$$

a phenomenon known as geometrical scaling [3].

Let us finish this section by recalling that the dipole-proton cross-section  $\sigma_{dP}$  is in the GBW model related to the unintegrated gluon distribution  $\varphi(x, k_T^2)$  [4, 2]:

$$\sigma_{dp}(x, r) = \frac{4\pi^2}{3} \int \frac{dk_T^2}{k_T^2} (1 - J_0(rk_T)) \alpha_s \varphi(x, k_T^2) \quad (7)$$

where

$$xG(x, Q^2) = \int^{Q^2} dk_T^2 \varphi(x, k_T^2). \quad (8)$$

The fact that  $\sigma_{dp}$  cross-section depends only on the combination  $r^2 Q_s^2(x)$  implies that

$$\varphi(x, k_T^2) = \varphi(k_T^2/Q_s^2(x)) \quad (9)$$

and strong coupling constant  $\alpha_s$  should be frozen.

## 2 Geometrical Scaling in pp Collisions

For pp collisions for low and moderate  $p_T$  one uses Gribov–Levin–Ryskin formula [5]

$$\frac{dN}{d\eta d^2p_T} = \frac{C}{p_T^2} \int d^2\vec{k}_T \alpha_s \varphi_1(x_1, \vec{k}_T^2) \varphi_2(x_2, (\vec{k} - \vec{p})_T^2). \quad (10)$$

Here  $x_{1,2}$  are gluon momenta fractions

$$x_{1,2} = e^{\pm y} p_T / W \quad \text{with} \quad W = \sqrt{s} \quad (11)$$

needed to produce a gluon of transverse momentum  $p_T$  and rapidity  $y$  (or pseudorapidity  $\eta$ ). Unfortunately formula (10) has been proven [6] only for the scattering of a dilute system on a dense one (*i.e.*  $x_1 \ll x_2$  or  $x_2 \ll x_1$ ). Despite that, one is often forced to use (10) in a region of  $x_1 \sim x_2$  where numerical studies show that it still works reasonably well (see e.g. Ref.[7]).

If unintegrated gluon densities scale according to Eq.(9) than also  $dN/d\eta d^2p_T$  should scale (provided we freeze  $\alpha_s$  and neglect possible energy dependence of constant  $C$ ). Therefore geometrical scaling for the multiplicity distribution in pp collisions [8, 10] states that particle spectra depend on one scaling variable

$$\tau = p_T^2 / Q_s^2(p_T, W) \quad (12)$$

where  $Q_s^2(p_T, W)$  is the saturation scale (1) at  $x_1 \sim x_2$  (11):

$$Q_s^2(p_T, W) = Q_0^2 (p_T / (W \times 10^{-3}))^{-\lambda} \quad (13)$$

where we have neglected rapidity dependence of  $x_{1,2}$ . Factor  $10^{-3}$  corresponds to the choice of the energy scale (arbitrary at this moment  $x_0$  in Eq.(1)). Hence

$$N_{\text{ch}}(W, p_T) \stackrel{\text{def}}{=} \left. \frac{dN_{\text{ch}}}{d\eta d^2p_T} \right|_W = \frac{1}{Q_0^2} F(\tau) \quad (14)$$

with  $Q_0 \sim 1$  GeV. Here  $F(\tau)$  is a universal function of  $\tau$ .

In order to examine the quality of geometrical scaling in pp collisions in Ref.[11] we have considered ratios  $R_{W_1/W_2}$

$$R_{W_1/W_2}(p_T) \stackrel{\text{def}}{=} \frac{N_{\text{ch}}(W_1, p_T)}{N_{\text{ch}}(W_2, p_T)}. \quad (15)$$

Here we shall discuss another way of establishing geometrical scaling, at least qualitatively. Note that if at two different energies  $W_1$  and  $W_2$  multiplicity distributions are equal

$$N_{\text{ch}}(W_1, p_T^{(1)}) = N_{\text{ch}}(W_2, p_T^{(2)}) \quad (16)$$

then this means that they correspond to the same value of variable  $\tau$  (12). As a consequence

$$p_T^{(1)2} \left( p_T^{(1)} / W_1 \right)^\lambda = p_T^{(2)2} \left( p_T^{(2)} / W_2 \right)^\lambda \quad (17)$$

for constant  $\lambda$ . Equation (17) implies

$$S_{W_1/W_2}^{p_T} \stackrel{\text{def}}{=} p_T^{(1)} / p_T^{(2)} = (W_1/W_2)^{\frac{\lambda}{2+\lambda}}. \quad (18)$$

Ratios  $S_{W_1/W_2}^{p_T}$  for pp non-single diffractive spectra measured by the CMS [9] collaboration at the LHC are plotted in the left panel of Fig. 1 together with the straight horizontal lines corresponding to the r.h.s. of Eq.(18) for  $\lambda = 0.27$ . We see approximate constancy of  $S_{W_1/W_2}^{p_T}$  over the wide range of  $N_{\text{ch}}$ . A small rise of  $S_{W_1/W_2}^{p_T}$  with decreasing  $N_{\text{ch}}$  corresponds to the residual  $p_T$ -dependence of the exponent  $\lambda$  [10].

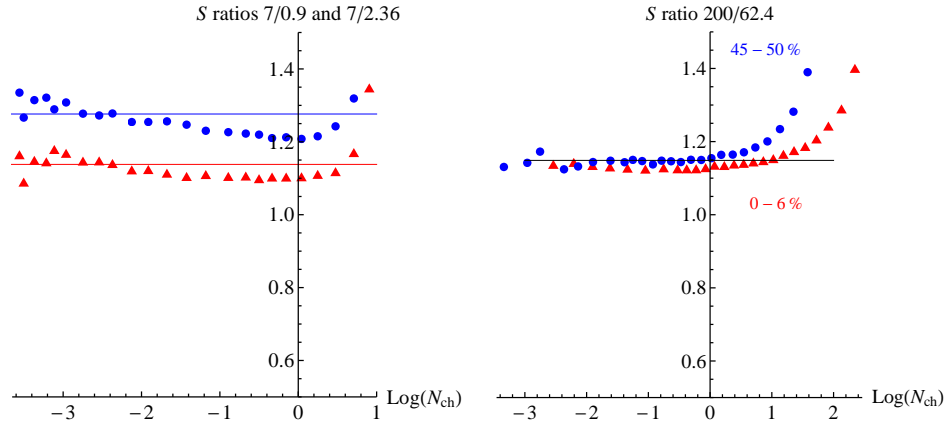


Figure 1: Plots of  $S_{W_1/W_2}^{p_T}$ . Left panel: CMS pp spectra for  $W_1 = 7$  TeV and  $W_2 = 0.9$  TeV (blue full circles) and 2.36 TeV (red triangles) in function of  $\log(N_{\text{ch}})$ . Right panel: PHOBOS Au-Au spectra for  $W_1 = 200$  GeV and  $W_2 = 62.4$  GeV in for two centrality classes: 0 – 5% (red triangles) and 45 – 50% (blue full circles) in function of  $\log(N_{\text{ch}})$ . Straight lines correspond to the r.h.s. of Eq. (18) for  $\lambda = 0.27$ .

We would like to advertise this method as a simple way of looking for GS in the  $p_T$  spectra in various reactions. An obvious advantage is that it is very easy to do: it requires only to interpolate one spectrum (here we have done this for  $W_1 = 7$  TeV). An obvious disadvantage consists in the fact that it is very difficult to attribute sensible error to the ratios  $S_{W_1/W_2}^{p_T}$ , so for quantitative purposes it is better to consider ratios  $R_{W_1/W_2}$ .

### 3 Onset of Geometrical Scaling in Heavy Ion Collisions

We have already briefly discussed the onset of GS in heavy ion collisions in Ref. [11]. This work is still in progress. If GS in HI collisions holds then, analogously to equation (18), we can form ratios of  $p_T$ 's corresponding to the same multiplicity (in the same centrality class  $c$ ) at two different energies  $W_{1,2}$ . In what follows we will neglect possible dependence of the saturation scale upon participant density [12] (for review see Ref. [13]).

Let us concentrate on the PHOBOS data for Au-Au [14, 15]. PHOBOS data cover asymmetric rapidity region  $0.2 < \eta < 1.4$  at two RHIC energies 64.2 and 200 GeV. The highest centrality bin for PHOBOS is only 45–50 %. In the right panel of Fig. 1 we plot ratio  $S_{200/62.4}^{p_T}$  for two centrality classes: 0–6 % and 45–50 %, together with a line corresponding to  $\lambda = 0.27$ . From Fig. 1 we might conclude that geometrical scaling works well also in heavy ion collisions. More detailed studies show, however, that dependence of the saturation scale on the participant densities may be crucial for proper description of the data.

### 4 Conclusions

In this note we have proposed a simple procedure to look for geometrical scaling in the  $p_T$  spectra, namely to construct ratios of transverse momenta corresponding to the same multiplicity. Based on this method one can qualitatively see GS in the CMS spectra [9] at the LHC energies and in HI collisions at RHIC [14, 15].

Many aspects of GS require further studies. Firstly, new data at higher energies (to come) have to be examined. Secondly, more detailed analysis including identified particles and rapidity dependence has to be performed. On theoretical side the universal shape  $F(\tau)$  has to be found and its connection to the unintegrated gluon distribution has to be studied. That will finally lead to perhaps the most difficult part, namely to the breaking of GS in pp.

Even more work is needed to understand GS quantitatively for HI collisions. Here the main question arises why hydrodynamic evolution preserves GS over a time of a few fermi until the final state particles are produced. Moreover, the behavior with energy of transverse participant densities for different centralities and for different rapidity regions require further studies. And finally  $A$  dependence of the saturation scale has to be confirmed. Here the new LHC data for different energies and different nuclei will open a new chapter, since the LHC energies are order of magnitude higher than those of RHIC.

### Acknowledgements

The author wants to thank the organizers of the Low- $x$  Meeting 2011 for putting together this very successful workshop. Special thanks are due to Larry McLerran and Andrzej Bialas for stimulating discussions. This work was supported in part by the Polish NCN grant 2011/01/B/ST2/00492.

### References

- [1] F. D. Aaron *et al.* [H1 and ZEUS Collaboration], JHEP **1001** (2010) 109
- [2] K.J. Golec-Biernat, M. Wusthoff, Phys. Rev. D **59** (1998) 014017, D **60** (1999) 114023.
- [3] A.M. Stasto, K.J. Golec-Biernat, J. Kwiecinski, Phys. Rev. Lett. **86** (2001) 596.
- [4] N. N. Nikolaev, B. G. Zakharov, Z. Phys. C **49** (1991) 607.
- [5] L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys. Lett. B **100** (1981) 173.
- [6] Y.V. Kovchegov, K. Tuchin, Phys. Rev. D **65** (2002) 074026.
- [7] E. Levin, A. H. Rezaeian, Phys. Rev. D **82** (2010) 014022.
- [8] L. McLerran, M. Praszalowicz, Acta Phys. Pol. B **41** (2010) 1917, B **42** (2011) 99.
- [9] V. Khachatryan *et al.* [CMS Collaboration], JHEP **1002** (2010) 041, Phys. Rev. Lett. **105** (2010) 022002, JHEP **1101** (2011) 079.
- [10] M. Praszalowicz, Phys. Rev. Lett. **106** (2011) 142002.
- [11] M. Praszalowicz, Acta Phys. Pol. **B42** (2011) 1557.
- [12] D. Kharzeev, M. Nardi, Phys. Lett. B **507** (2001) 121.
- [13] L. McLerran, Acta Phys. Pol. B **41** (2010) 2799.
- [14] B. B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **94** (2005) 082304.
- [15] B. Alver *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **96** (2006) 212301.